THE LOW FLOW EXTREMES IN A SMALL BESKYDY MTS. EXPERIMENTAL WATERSHED

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Experimental watershed

An experimental forested watershed Červik (1.85 km²)[1] is situated in the Beskydy Mountains, Czech Republic in the upper basin of Odra river. The hydrological research started there on November 1st 1953 and is continued till today. After this period three times accelerated forest renewal with cut in stripes between 1966-98 followed. The watershed suffered heavily from air pollution especially in the years 1970-1990 (Biba *et al.*, 2007). For the low flow extremes estimation the daily run offs are divided on three groups of years:

− 1954–1983

− 1966–1995

− 1987–2007.

It is assumed that each group consists of the homogenous low flow data.

Bivariate Generalized Pareto Distribution

For estimation low flows deficit and duration extremes the Bivariate Generalized Pareto Distribution (BGPD) $H(\chi \cdot t)$ is applied (Jakubowski, 2006). According to Tajvidi (1996)

$$
\begin{aligned}
H(x, t) &= 1 - \Pr\left((D_M, T_M) \nleq (x, t)\right) \\
&= 1 - \left(\frac{\overline{F}_d^p(x) + k\overline{F}_d^{p/2}(x)\overline{F}_t^{p/2}(t) + \overline{F}_t^p(t)}{\overline{F}_d^p(0) + k\overline{F}_d^{p/2}(0)\overline{F}_t^{p/2}(0) + \overline{F}_t^p(0)}\right)^{\frac{1}{p}},
\end{aligned}
\tag{1}
$$

where

− $D_M = \max(D_1, \dots, D_n)$ is maximal low flow deficit;

− $T_M = \max(T_1, \dots, T_n)$ is maximal low flow duration;

$$
\begin{aligned}
\overline{F}_d(x) &= \left(1 - \alpha_d \kappa_d (b_d + x)\right)^{1/\kappa_d}, \\
\overline{F}_t(t) &= \left(1 - \alpha_t \kappa_t (b_t + t)\right)^{1/\kappa_t}
\end{aligned}
\tag{2}
$$

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[1] The daily runoffs are obtained from Forestry and Game Management Research Institute Jíloviště, Czech Republic

– the distributions functions $F_d(x) = 1 - \overline{F}_d(x), \; F_t(t) = 1 - \overline{F}_t(t)$ are Univariate Generalized Pareto Distributions and they describe each of the indices separately;

$$0 \leq k \leq 2(p-1), \; p \geq 2, \; \kappa_d, \kappa_t \in \left(-\tfrac{1}{2}; 0\right), \; \alpha_d, \alpha_t > 0$$

– b_d, b_t are the shift parameters for deficit and duration suitably belongs to the family of Bivariate Generalized Pareto Distributions (BGPD) with positive support.

Estimation is performed for two dimensional observations of low flow deficits and durations. To estimate the distributions of extreme indices the above defined BGPD (1) is applied. As it can be seen above, the BGPD depends on eight parameters. Six of them are connected with two 3 – parameter UPGDs. The final two p, k are related to the form of the two dimensional formula. For the estimation the following method is applied:

1. For a given pair of shift parameters (b_d, b_t) the four of them $\hat{\alpha}_d, \hat{\kappa}_d, \hat{\alpha}_t, \hat{\kappa}_t$ are estimated by the maximum likelihood method, for each of the one-dimensional indices separately. Each sequence of the index observations is decreased by a shift parameter then the standard maximum likelihood method is applied. The goodness of fit for each of the UPGDs is achieved by using the χ^2 test. For further estimation the pairs of the shift parameters which do not reject the goodness of fit tests are considered only. The two last $\hat{p}, \hat{k}$ are estimated by the BGPD using the maximum likelihood method as well. The chosen shift parameters sequences are equal to the successive ordered low flow deficits and durations.

2. The best shift pair $(b_d \cdot b_t)$ is chosen by the maximization of the correlation coefficient. This assumption is made because of the non-homogeneity of observed low flows. Upon observation, it is suffice to analyze the nonlinear dependence between deficit amounts and durations (for instance see Fig. 1 below). Along with the increase of the deficit duration the volumes grow much quicker – this is clearly visible for short durations. By taking the maximum correlation coefficient the homogeneity of processed low flow observations is stabilized. All computations are carried out for shift pairs with connected estimators $(\hat{\kappa}_d, \hat{\kappa}_t)$ in the interval $\left(-\tfrac{1}{2}; 0\right)$ only. Other pairs, where at least one $\hat{\kappa}$ stays outside the interval $\left(-\tfrac{1}{2}; 0\right)$, are omitted.

3. The goodness of fit of the estimated one-dimensional marginal distributions of extreme annual or seasonal index extremes is obtained by λ – Kolmogorov goodness of fit test.

Results and conclusions

The estimation results for the years 1954-83 are shown in Figures 1-3. They present estimated two dimensional probability plots of extreme low flow deficit amounts and durations. In Figure 1 asterisks denote the observed annual or summer maximums of deficits or durations, crosses refer to other significant observed droughts. Straight
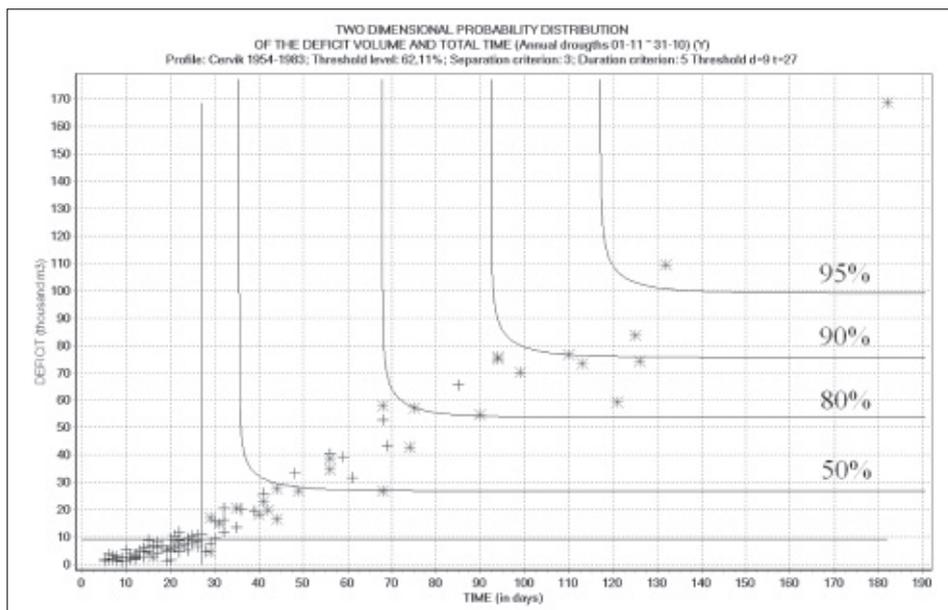

Figure 1. An annual low flows – fitting into the Bivariate Generalized Pareto Distribution
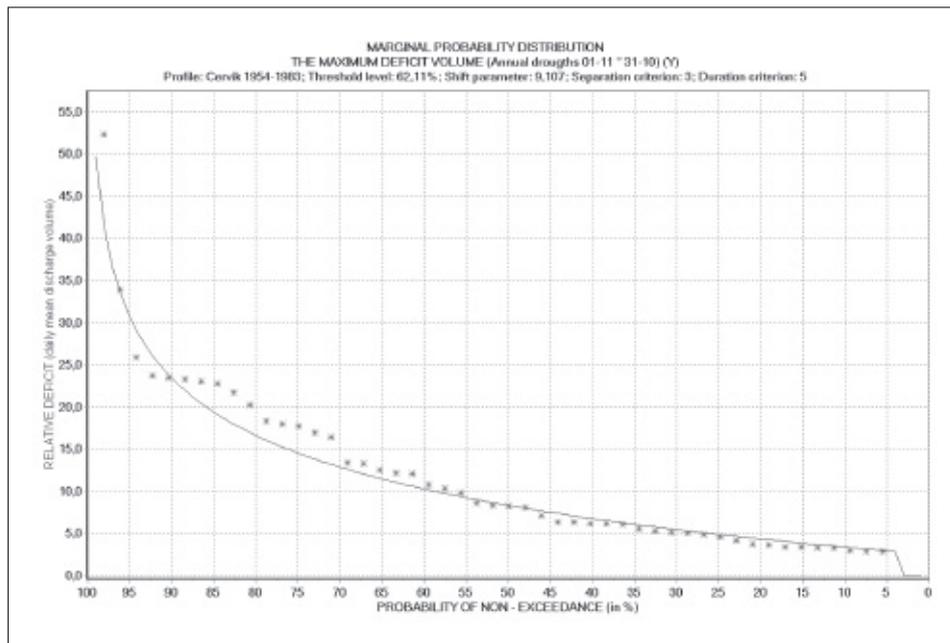
Figure 2. An annual low flows – fitting into marginal distributions of the low flow maximum deficit volume
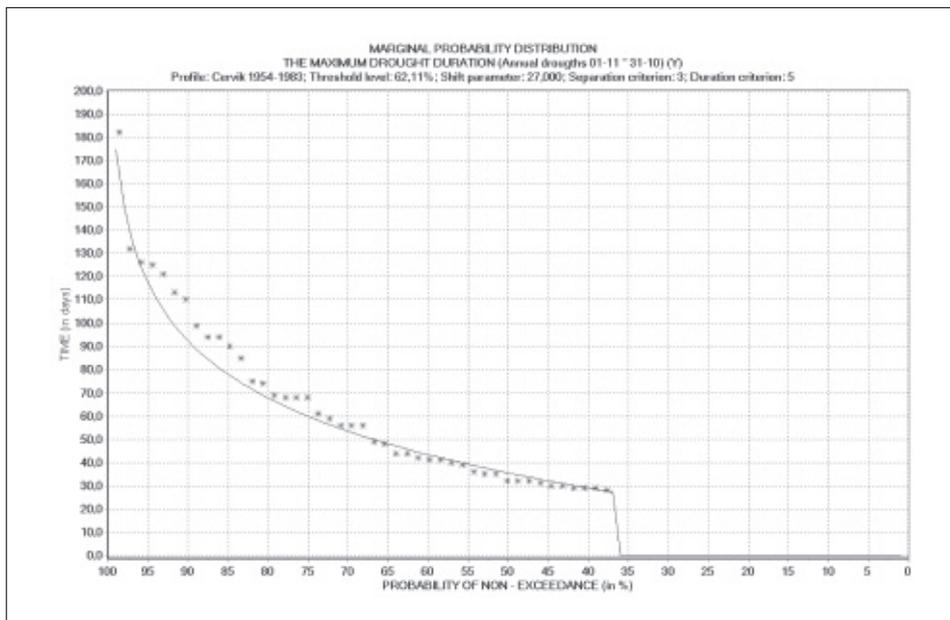

Figure 3. An annual low flows – fitting into marginal distributions of the low flow maximum duration

lines depict the estimated best shift parameters. The quantile curves – constant value probability lines determine the areas laying left or below them whose estimated probabilities of non-exceedance (1) are equal to 50, 80, 90 and 95%.

Taking the marginal distributions of the estimated BGPD (1) one can compute the probability of each of the examined maximum values of the indices. In Figures 2-3 the probabilities of the non-exceedance of the low flow extreme deficit amounts and durations are presented. Because of the positive probability values of the marginal distribution $H(\chi \cdot \infty)_{\chi=0}$ and $H(\infty, t)_{t=0}$ the one-dimensional distributions of low flow indices have a discontinuous probability jump set at the estimated shift parameter.

The graphs for estimation results for the years 1966-95 and 1978-2007 are looking out similarly.
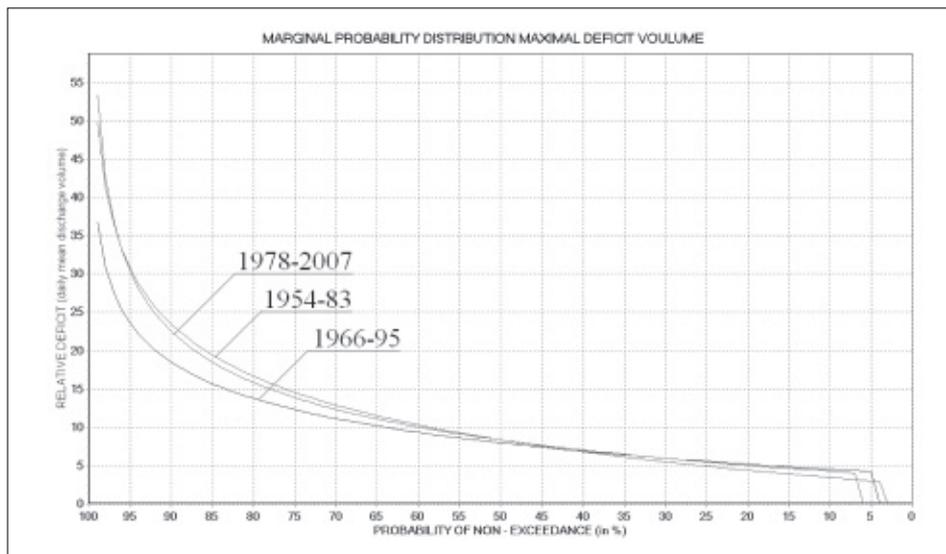
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Figure 4. Comparison of estimated univariate distributions of the maximum low flow deficits
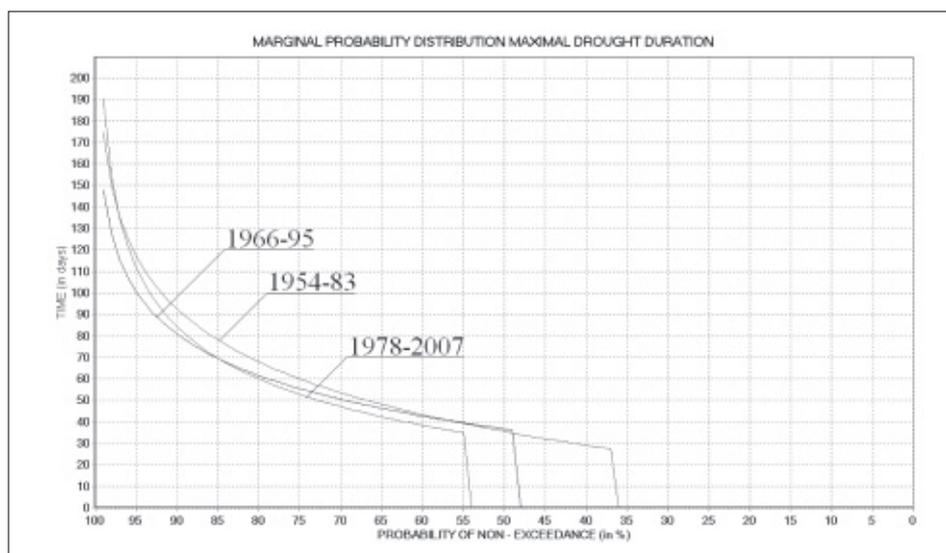

Figure 5. Comparison of estimated univariate distributions of the maximum low flow durations

The comparison of the estimated univariate distributions is shown in Figures 4-5. Specially the maximum deficit distributions clearly show that during the researched years there were deep changes in the watershed environment. For the low flows point of view years 1966- 95 are not similar to the previous period 1954-83. It can be also seen that in last years the conditions are slowly returning to earlier environmental state.

References

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